Balanced Growth and Structural Change

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Abstract

This paper reconciles two stylized facts that characterize modern economic growth, balanced growth, and structural change, in a context where the factor intensities differ. I extend the neoclassical growth model to two sectors with different factor intensities, and I derive the dynamics of the sectoral TFPs that ensure aggregate balanced growth. I derive the condition on the TFP growth such that balanced growth is consistent with structural change. The condition of balanced growth in a two-sector model with different factor intensities is that the aggregate TFP growth minus the wage growth weighted by the aggregate labor share is constant. In this framework, structural change occurs through two channels. The first is the change in the sectoral TFP ratio and the second is the change in the relative cost of factors. The empirical analysis confirms that the model replicates the stylized facts aforementioned.
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Introduction

For nearly 150 years, real GDP per capita in the United States has grown at a remarkably steady average rate of around 2% per year, while the ratio of physical capital to output has remained nearly constant (Jones (2016)). These facts are referred to as *Kaldor facts* (see Kaldor (1961), Denison (1974), Homer and Sylla (1991), Barro (2004)). However, along this balanced pattern, there are systematic changes in the sectoral contribution to the aggregate value-added (see Chenery (1960), Kuznets (1957), (1973), Kongsamut et al. (2001)). In 1870, agriculture employed 40% of all Americans. Agriculture accounted for only 4% of employment 100 years later (Kongsamut et al. (2001)). Herrendorf et al. (2021) documented that, while the value-added share and the employment share of the goods sector have decreased since 1947, the value-added share and the employment share of the services sector have increased. This sectoral reallocation of factors and value-added across sectors is referred to as *Kuznets facts*.

Two literature lines reconcile Kaldor facts and Kuznets facts[1]. The first line of the literature considers nonhomothetic preferences consistent with Engel's law. In this case, as an economy grows, the marginal rate of substitution between different goods changes, resulting in structural change[2]. The second line of the literature emphasizes the role of technological differences across industries[3]. Ngai and Pissarides (2007) takes into account homothetic preferences and sectoral production functions with varying TFP growth rates. The uneven TFP growth rate drives structural change in this model. Acemoglu and Guerrieri (2008) (hereafter AG) consider a multi-sector framework variant in which the factor intensities of the sectoral production functions differ. Technological change has a double effect on structural change in this framework. Not only do differences in the sectoral TFP growth rates

[1]Gabardo et al. (2017) provide an overview of some of the key works in modern growth theory and evaluate the incorporation of structural change into economic growth analysis.

[2]See for example Matsuyama (1992), Echevarria (1997), Laitner (2000), Caselli and Coleman (2001), Gollin et al. (2002), Foellmi and Zweimüller (2008), Duarte and Restuccia (2010), Boppart (2014) etc.

[3]See Baumol (1967), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Herrendorf et al. (2021), Alvarez-Cuadrado et al. (2017), etc.

affect differently the sectoral value-added, but the implied capital accumulation also has an unequal effect across sectors[4].

AG model is not, however, consistent with the Kaldor facts. Indeed, in their framework, balanced growth exists only asymptotically. Furthermore, only one sector survives asymptotically. As a result, balanced growth and structural change do not coexist.

This paper develops a variant of AG model where balanced growth and structural change are consistent. The framework is the neoclassical growth model with two sectors. As in AG, the two sectors have different factor intensities (factor proportions). Unlike AG, the framework does not constrain a constant growth of technological change. Rather, I derive the dynamics of the sectoral TFPs that ensure aggregate balanced growth. I derive the condition on the TFP growth such that balanced growth is consistent with structural change.

The condition of balanced growth in a two-sector model with different factor intensities is such that the weighted sum of the sectoral TFP growth is constant. The corresponding weights depend on the TFP levels. The two-sector framework's balanced growth differs from neoclassical balanced growth in two ways. First, the sectoral TFP growth rates are not constant and vary according to sectoral TFP levels. Second, the growth rate of wages is not necessarily zero.

In this framework, structural change occurs through two channels. The first is the change in the sectoral TFP ratio and the second is the change in the relative cost of factors. In the case where goods and services are gross complement, if the relative cost of labor rises, then the value-added share of the labor-intensive sector increases, and vice versa. The opposite occurs if the sectors are gross substitute. The contribution of the relative cost of factors to structural change is implied by the fact that the relative increase in the cost of labor has a greater impact on the labor-intensive sector and the marginal cost of factor increases in relative terms.

With different factor intensities, balanced growth with structural change is not consistent

[4] In AG the effect of changes in capital stock on structural change is named capital deepening effect.

with constant sectoral TFP growth. If the augmented TFP growth[5] is identical across sectors and sectoral TFP grows at a constant rate, aggregate value-added grows at a constant rate as well, but the value-added share of sectors remains constant. Aside from this specific case, there is no other balanced growth with constant sectoral TFP growth, and any other balanced growth features an unavoidable structural change pattern.

I calibrate the model to the US economy and estimate the TFP so that the price ratio of the goods and services sectors matches the data and aggregate value-added growth is constant. Although the sectors have different production functions in this quantitative analysis, the aggregate value-added is balanced, and the value-added of the goods sector decreases relative to the value-added of the services sector. The model accounts for 50% of the structural change. The corresponding sectoral TFP dynamic is not constant, but rather quasi-constant. Although linear changes in the log of TFP are incompatible with a balanced economy undergoing structural change, minor non-linearity is sufficient to reconcile balanced growth and structural change.

The paper is organized as follows. In the first section, I present a two-sector economy framework with different factor intensities. I describe the equilibrium factor behavior in the second section. I characterize the TFP dynamic that generates balanced growth in the third section. The fourth section examines the condition under which balanced growth is compatible with structural change. Finally, I simulate the balanced growth path in the last section and find that the behavior of the sectoral value-added shares along the balanced growth path is consistent with the data.

1. Two-Sector Economy Framework

In this section, I develop a framework of two-sector economy. The framework is a neoclassical general equilibrium growth model with two sectors, goods and services. The factor

[5]The augmented growth rate is coined in AG and represents the TFP growth rate divided by the labor intensity in the Cobb-Douglas production function with a constant return-to-scale.

intensities in the goods sector and the services sector differ. I use a production function with a decreasing return-to-scale. The primary reason is that such a framework is more tractable. Indeed, the decreasing return-to-scale framework clearly defines the quantities of factors and the value-added. However, I extend the decreasing return-to-scale framework's results to the constant return-to-scale case.

1.1. Preferences

The framework features a representative household. The lifetime consumption yields a utility represented by the following function:

$$U = \int_0^\infty \mathrm{e}^{-\varrho t} \frac{C(t)^{1-\sigma}}{1-\sigma} \mathrm{d}t \tag{1}$$

where $C(t)$ is the consumption of period t. The total income is the sum of the capital income $r(t)K(t)$, the labor income $w(t)N(t)$ and the firms profit $\Pi(t)$. The total expenditure is the sum of the consumption expenditure $P(t)C(t)$ and the capital expenditure $P(t)\dot{K}(t)$. $P(t)$ is normalized to one. Formally, the budget constraint is:

$$\dot{K}(t) + C(t) = r(t)K(t) + w(t)N(t) + \Pi(t) \tag{2}$$

1.2. Firms

The economy is populated by three firms. The first firm produces goods denoted by Q_1; the second firm produces services denoted by Q_2. The third firm is an aggregate producer and combines goods and services to produce the gross output denoted by Q. The production of goods and services uses capital and labor according to the following function.

$$Q_s(t) = \epsilon_s(t) K_s(t)^{\alpha_s} N_s(t)^{\gamma-\alpha_s}, \qquad s \in \{1, 2\} \tag{3}$$

where ϵ_s and α_s are the total factor productivity (TFP), and the capital intensity of firm s, respectively. K_s and N_s represent the amount of capital and labor used by the firm s, respectively.

The production function is characterized by a decreasing return-to-scale, in other words, $\alpha_s < \gamma < 1$. Let $r(t)$ and $w(t)$ be the interest rate and the wage rate at period t, respectively. The firms are price takers, and their respective problems are formalized as follows:

$$\max_{N_s(t),K_s(t)} P_s(t)\epsilon_s(t)K_s(t)^{\alpha_s}N_s(t)^{\gamma-\alpha_s} - w(t)N_s(t) - r(t)K_s(t), \qquad s \in \{1,2\} \tag{4}$$

where P_s denotes the price of the output of the firm s.

The aggregate producer uses Q_1, and Q_2, the value-added in the goods and the services sectors respectively, to produce the aggregate output Q. Her production function is the following:

$$Q(t) = (\psi Q_1(t)^\rho + (1-\psi)Q_2(t)^\rho)^{\frac{1}{\rho}} \tag{5}$$

ψ is the weight of goods and $1-\psi$ is the weight of services in the production of the aggregate output. ρ is the substitution parameter such that the elasticity of substitution between goods and services equals $\frac{1}{1-\rho}$. If $\rho < 0$, then goods and services are gross complement, otherwise they are gross substitute. The aggregate producer maximizes her profit by taking the prices of the inputs and the output as given.

$$\max_{Q_1(t),Q_2(t)} P(t)\left(\psi Q_1(t)^\rho + (1-\psi)Q_2(t)^\rho\right)^{\frac{1}{\rho}} - P_1(t)Q_1(t) - P_2(t)Q_2(t) \tag{6}$$

1.3. Equilibrium

Let $K(0)$ be the initial stock of capital. A competitive equilibrium is a set of prices $\{P_1(t), P_2(t), r(t), w(t)\}$, a total consumption $C(t)$, a stock of capital $K(t)$, the sectoral value-added $Y_s(t)$, and the aggregate value-added $Y(t)$, such that:

(i) Given the price of the gross output, $C(t)$ and $\dot{K}(t), t \in \mathbb{N}$ maximize the household lifetime utility.

(ii) Given the prices of the factors of production, $N_s(t)$, and $K_s(t)$ maximize the firm s profit; $Q_1(t)$, and $Q_2(t)$ maximize the aggregate producer's profit.

(iii) The capital market clears

$$K_1(t) + K_2(t) = K(t) \tag{7}$$

(iv) The labor market clears

$$N_1(t) + N_2(t) = N(t) \tag{8}$$

(v) The aggregate output market clears

$$\dot{K}(t) + C(t) = Q(t) \tag{9}$$

2. Model Analysis

In this section, I calculate the factor demands by sector. I compute the optimal value-added as well as the prices for each sector. Finally, I compute the value-added and employment shares of the goods sector and the services sector.

Factor demands are determined by the following equations:

$$w(t)N_s(t) = (\gamma - \alpha_s)P_s(t)^{\frac{1}{1-\gamma}}\epsilon_s(t)^{\frac{1}{1-\gamma}}\left(\frac{\alpha_s}{r(t)}\right)^{\frac{\alpha_s}{1-\gamma}}\left(\frac{\gamma-\alpha_s}{w(t)}\right)^{\frac{\gamma-\alpha_s}{1-\gamma}}$$

$$r(t)K_s(t) = \alpha_s P_s(t)^{\frac{1}{1-\gamma}}\epsilon_s(t)^{\frac{1}{1-\gamma}}\left(\frac{\alpha_s}{r(t)}\right)^{\frac{\alpha_s}{1-\gamma}}\left(\frac{\gamma-\alpha_s}{w(t)}\right)^{\frac{\gamma-\alpha_s}{1-\gamma}}$$

The optimal real value-added in the goods sector and the services sector is defined by the following equation:

$$Q_s(t) = P_s(t)^{\frac{\gamma}{1-\gamma}}\epsilon_s(t)^{\frac{1}{1-\gamma}}\left(\frac{\alpha_s}{r(t)}\right)^{\frac{\alpha_s}{1-\gamma}}\left(\frac{\gamma-\alpha_s}{w(t)}\right)^{\frac{\gamma-\alpha_s}{1-\gamma}}$$

where $s = 1, 2$.

The aggregate output price $P(t)$ is normalized to one; therefore, we have the following equation:

$$P(t) = \left[\psi^{\frac{1}{1-\rho}} P_1(t)^{-\frac{\rho}{1-\rho}} + (1-\psi)^{\frac{1}{1-\rho}} P_2(t)^{-\frac{\rho}{1-\rho}}\right]^{-\frac{1-\rho}{\rho}} = 1$$

The first-order condition of the aggregate producer's problem is that the marginal rate of transformation equals the relative prices. I compute the relative prices by combining this optimal condition with the sectoral value-added derived above.

$$\frac{P_1(t)}{P_2(t)} = \left(\frac{\psi}{1-\psi}\right)^{\frac{1-\gamma}{1-\gamma\rho}} \left[\theta \frac{\epsilon_1(t)}{\epsilon_2(t)} \left(\frac{w(t)}{r(t)}\right)^{\alpha_1-\alpha_2}\right]^{-\frac{1-\rho}{1-\gamma\rho}} \tag{10}$$

where $\theta = \frac{\alpha_1^{\alpha_1}(\gamma-\alpha_1)^{\gamma-\alpha_1}}{\alpha_2^{\alpha_2}(\gamma-\alpha_2)^{\gamma-\alpha_2}}$. When the sectors have the same factor intensities, only the TFP ratio determines the relative prices. When the intensities of the factors of production differ, the ratio of the cost of the factors of production also affects the relative prices. The relative price of the labor-intensive sector rises as the relative cost of labor increases.

Next, I determine the value-added share and the employment share of the goods sector and the services sector. Let us denote by $\Delta = \frac{P_2 Q_2}{P_1 Q_1}$, the nominal value-added of the services sector relative to the goods sector. This ratio depends on the ratio of the TFPs and the relative cost of factors.

$$\Delta(t) = \left(\frac{1-\psi}{\psi}\right)^{\frac{1}{1-\gamma\rho}} \left(\frac{1}{\theta}\frac{\epsilon_2(t)}{\epsilon_1(t)} \left(\frac{w(t)}{r(t)}\right)^{\alpha_2-\alpha_1}\right)^{\frac{\rho}{1-\gamma\rho}}$$

The prices of goods and services are given by:

$$P_1(t) = \psi^{\frac{1}{\rho}} \left(\Delta(t) + 1\right)^{\frac{1-\rho}{\rho}}; \qquad P_2(t) = (1-\psi)^{\frac{1}{\rho}} \left(\Delta(t)^{-1} + 1\right)^{\frac{1-\rho}{\rho}} \tag{11}$$

Because the TFP reduces the marginal cost of production, an increase in the TFP of a given sector lowers its relative price.

The employment share of the sector s is the amount of labor used in sector s relative to the total labor $\frac{N_s}{N_1+N_2}$. Let S_1^N and S_2^N denote the employment share of the goods sector and the services sector, respectively.

$$S_1^N(t) = \frac{1}{1+\frac{\gamma-\alpha_2}{\gamma-\alpha_1}\Delta(t)}; \qquad S_2^N(t) = \frac{1}{1+\frac{\gamma-\alpha_1}{\gamma-\alpha_2}\Delta(t)^{-1}} \tag{12}$$

The value-added share of the sector s is the proportion of the value-added of the sector s in the total value-added $\frac{P_sQ_s}{P_1Q_1+P_2Q_2}$. Let S_1^Q and S_2^Q denote the value-added share of the goods sector and the services sector, respectively.

$$S_1^Q(t) = \frac{1}{1+\Delta(t)}; \qquad S_2^Q(t) = \frac{1}{1+\Delta(t)^{-1}} \tag{13}$$

If goods and services are complement, then a relative increase in the TFP of a given sector reduces its employment and value-added shares by lowering its relative price. When the sectors are substitute, the opposite occurs.

3. Balanced Growth

In this section, I derive the condition of balanced growth. According to neoclassical theory, balanced growth occurs when all factors grow at the same rate. However, such a definition is too narrow and can not be applied to an economy with structural change. I then use the definition of Kongsamut et al. (2001) stated as follows:

Definition 1. *A balanced growth path is a trajectory along which the real interest rate is constant*[6]*.*

Balanced growth is characterized by a constant interest rate. This definition implies that the major economic aggregates, such as consumption, savings, and total value-added,

[6]This definition of balanced growth is referred to as general balanced growth path in Herrendorf et al. (2021), and constant growth path in Acemoglu and Guerrieri (2008)

grow at a constant rate. However, the rate of growth in sectoral output does not have to be constant along the balanced growth path. Indeed, the optimal path of consumption is determined by the Euler equation.

$$\hat{C}(t) = \frac{1}{\sigma}(r(t) - \varrho) \tag{14}$$

The *hat* sign represents the growth rate. Regarding the Euler equation, a constant interest rate implies that the total consumption grows at a constant rate. When the interest rate is constant, both savings and consumption grow at the same rate (see Moll et al. (2019)). Therefore, savings grow at a constant rate along the balanced growth path. According to equation (9), the total value-added growth is also constant along the balanced growth path.

Hereafter, I drop the time symbol to simplify the notation. The next proposition states the necessary and sufficient condition of balanced growth of an economy with two sectors and different factor intensities.

Proposition 1. *For any initial condition $K(0) > 0$, the equilibrium allocation is on a balanced growth path for all $t > 0$ if and only if the sectoral TFP sequences satisfy the condition that*

$$S_1^Q \hat{\epsilon}_1 + S_2^Q \hat{\epsilon}_2 - \Big(S_1^Q(\gamma - \alpha_1) + S_2^Q(\gamma - \alpha_2)\Big)\, \hat{w} \text{ is constant} \tag{15}$$

where

$$\hat{w} = \frac{\rho(1-\gamma)\left(S_1^N \hat{\epsilon}_1 + S_2^N \hat{\epsilon}_2\right) + (1-\rho)\Big(S_1^Q \hat{\epsilon}_1 + S_2^Q \hat{\epsilon}_2\Big)}{\rho(1-\gamma)\left(1 - S_1^N \alpha_1 - S_2^N \alpha_2\right) + (1-\rho)\Big(1 - S_1^Q \alpha_1 - S_2^Q \alpha_2\Big)} \tag{16}$$

Proof. See Appendix A □

Proposition 1 states the condition of balanced growth in a two-sector growth model with different factor intensities in equation (15), as well as the wage growth rate in equation (16). Balanced growth implies that the aggregate TFP growth minus the wage growth weighted by the aggregate labor share is constant. In addition, wage growth is the weighted sum of the sectoral TFP growth rates. The aggregate TFP growth rate is calculated as the weighted mean of the sectoral TFP growth rates, where the weights represent the sectoral value-added share. The aggregate labor share is the weighted mean of the sectoral labor share, where the weights are also the sectoral value-added share.

The balanced growth path condition in this framework differs from the neoclassical balanced growth path condition in two ways. First, the sectoral TFP growth rates are not constant and vary according to sectoral TFP levels. Indeed the value-added share and the employment share are functions of the sectoral TFP levels. Second, the cost of labor is not necessarily constant along the balanced growth path. However, the condition includes balanced growth paths with constant wages. Corollary 1 characterizes the balanced growth path with a constant wage rate.

Corollary 1. *For any initial condition $K(0) > 0$, the equilibrium allocation is on a balanced growth path with a constant wage rate for all $t > 0$ if and only if the sectoral TFP sequences satisfy the condition that*

$$\begin{cases} \hat{\epsilon}_1 = \frac{\kappa\left(\frac{(1-\gamma)\rho}{1-\gamma\rho} S_1^N + \frac{1-\rho}{1-\gamma\rho} S_1^Q - 1\right)}{S_1^Q - S_1^N} \\ \hat{\epsilon}_2 = \frac{\kappa\left(\frac{(1-\gamma)\rho}{1-\gamma\rho} S_1^N + \frac{1-\rho}{1-\gamma\rho} S_1^Q\right)}{S_1^Q - S_1^N} \end{cases} \tag{17}$$

where κ is a real number s.t. $\rho\kappa < 0$.

If goods and services are complement, balanced growth with a constant wage rate results in faster TFP growth in the labor-intensive sector. If the sectors are substitute, then the balanced growth path implies that the capital-intensive sector's TFP grows faster.

The next proposition shows the sectoral dynamics of prices, factors of production, and the value-added.

Proposition 2. *Along the balanced growth path, the sectoral aggregate growth rates are determined by:*

$$\hat{P}_1 = \frac{1-\rho}{1-\gamma\rho} S_2^Q \left(\hat{\epsilon}_2 - \hat{\epsilon}_1 + (\alpha_2 - \alpha_1)\hat{w}\right) \tag{18}$$

$$\hat{P}_2 = \frac{1-\rho}{1-\gamma\rho} S_1^Q \left(\hat{\epsilon}_1 - \hat{\epsilon}_2 + (\alpha_1 - \alpha_2)\hat{w}\right) \tag{19}$$

$$\hat{w} + \hat{N}_s = \hat{K}_s = \hat{P}_s + \hat{Q}_s = \frac{1}{1-\gamma}\left(\hat{P}_s + \hat{\epsilon}_s - (\gamma - \alpha_s)\hat{w}\right) \tag{20}$$

Proof. See Appendix A □

Equations (18) and (19) represent the growth rates of prices of goods and services, respectively. Prices rise more quickly in the slowest TFP sector. The increase in wages accelerates price increases in the labor-intensive sector while slowing price increases in the capital-intensive sector.

Equation (20) shows that the growth rates of the sectoral labor income, the capital income, and the value-added are identical and depend on the growth rate of the sectoral prices, TFPs, and the change in the wage rate. While the rise in the TFP and the output prices increases the income and the value-added, the rise in wages has the opposite effect.

The condition of balanced growth stated in Proposition 1 is derived with decreasing return-to-scale (DRS) production function in goods and services sectors. The next purpose is to derive the condition of balanced growth with constant return-to-scale (CRS). To that end, I use Lemma 1 to extend DRS to CRS. According to Lemma 1, the limit and function symbols commute in a metric space for a continuous mapping.

Let's recall that γ is the span of control of the production function of goods and services. $\gamma < 1$ represents DRS and $\gamma = 1$ represents CRS. Let's denote by $\mathcal{F}_\gamma$, a framework characterized by a set of continuous functions in a metric space (X, d). Let's also denote by Φ_γ the outcome of the framework $\mathcal{F}_\gamma$. The outcome of the framework represents the set of the framework's analytical results. If the limit Φ_1 of Φ_γ exists when the span of control tends to unity, then Φ_1 is the analytical solution of the framework with CRS. Proposition 3 states this result more formally.

Proposition 3. *Let γ be a real number smaller than one. Let $\mathcal{F}_\gamma$ be a framework characterized by a set of continuous functions in a metric space (X, d), where the production* *function has the return-to-scale γ. Let Φ_γ be the set of analytical results of $\mathcal{F}_\gamma$, $\forall\gamma < 1$ s.t. the mapping $\mathcal{M} : \mathcal{F}_\gamma \longmapsto \Phi_\gamma$ is continuous.*

If $\Phi_1 = \lim_{\gamma\to 1} \Phi_\gamma$ exists, then the analytical results of $\mathcal{F}_1$, the variant of $\mathcal{F}_\gamma$ where the production function has a constant return-to-scale, is Φ_1.

Proof. See Appendix B □

The outcome of a model with CRS is the limit of the outcome of a model with DRS when $\gamma \to 1$. Proposition 3 comes in handy when the parameterized framework is easier to analyze. Indeed, a DRS framework is more tractable than a CRS case. The proposition demonstrates how to transition from a framework with DRS to a CRS case.

In light of Proposition 3, I derive the condition of balanced growth in an economy with CRS by using the results of the DRS case. In this case, the outcome Φ_γ represents the growth rate of the aggregate output. With DRS, $\Phi_\gamma = \frac{S_1^Q \hat{\epsilon}_1 + S_2^Q \hat{\epsilon}_2 - \left(S_1^Q(\gamma-\alpha_1) + S_2^Q(\gamma-\alpha_2)\right)\hat{w}}{1-\gamma}$, where $\hat{w}$ is given in equation (16).

According to Proposition 3, if Φ_1 exists, then Φ_1 is the growth rate of the aggregate output with CRS. Appendix B shows that Φ_1 exists; therefore, the economy with CRS is on the balanced growth path if Φ_1 is constant. Proposition 4 states it formally.

Proposition 4. *Let us assume that $\gamma = 1$.*

For any initial condition $K(0) > 0$, the equilibrium allocation is on a balanced growth path for all $t > 0$ if and only if the sectoral TFP sequences satisfy the condition that

$$S_1^N \hat{\epsilon}_1 + S_2^N \hat{\epsilon}_2 - \left(S_1^N(1-\alpha_1) + S_2^N(1-\alpha_2) + \frac{1-\rho}{\rho}\right)\hat{w} \text{ is constant} \tag{21}$$

Proof. See Appendix B □

The maximum theorem guarantees that the mapping $\mathcal{M}$ that transforms the consumer and the producer problem into the equilibrium output growth is continuous, in order to apply Proposition 3.

Balanced growth, in the CRS case, implies that the aggregate TFP growth minus the wage growth weighted by the aggregate labor share is constant. In this case, the aggregate TFP growth is the weighted mean of sectoral TFP growth rates, where the weights are the employment shares. The aggregate labor share is the weighted mean of the sectoral labor shares adjusted for the elasticity of substitution between goods and services. The employment share and the wage growth rate are given in equation (12) and (16) respectively, where γ is equal to one.

4. Structural Change

In this section, I investigate structural change along the balanced growth path. The definition of structural change is given below.

Definition 2. *Structural change implies that the value-added share or similarly the employment share increases in one sector and decreases in another sector*[7].

According to equations (12) and (13), structural change occurs through two channels. The first channel is the TFP ratio. Increases in the relative TFP lower relative prices. Indeed, an increase in TFP in a given sector leads to a decrease in the marginal cost of production of that sector.

The second channel is the relative cost of factors. When the relative cost of a given factor rises, so does the relative price of the sector that is more intensive on that factor. In other words, if labor becomes more expensive than capital, the labor-intensive sector's relative price rises. Similarly, as the cost of capital rises, so does the relative price of the capital-intensive sector. Changes in relative prices affect the relative value-added share. This second channel exists only if the sectors' factor intensities differ. Indeed, with identical factor intensities, changes in the relative factor cost affect evenly the sectoral marginal cost, hence the price ratio remains unchanged. However, with different factor intensities, the

[7]See for example Herrendorf et al. (2021), Herrendorf et al. (2014), Kongsamut et al. (2001)

marginal cost rises faster in the sector that is more intensive on the factor with rising cost in relative terms.

Next, I study the consistency of structural change and balanced growth. The following proposition describes the condition of balanced growth with and without structural change.

Proposition 5. *Let $K(0)$ be the initial stock of capital. The equilibrium allocation is on a balanced growth path without structural change for all $t > 0$ if and only if the sectoral TFP sequences satisfy the following conditions:*

(i) $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$ are both constant

(ii) $\frac{\hat{\epsilon}_1}{1-\alpha 1} = \frac{\hat{\epsilon}_2}{1-\alpha 2}$.

Any other balanced growth path features necessarily a structural change pattern.

Proof. See Appendix A □

The quantity $\frac{\hat{\epsilon}_s}{1-\alpha_s}$ is referred to as the augmented rate of technological change in AG. Indeed, since the two sectors have different capital intensities, the technological change will be augmented by the differential rates of capital accumulation in the two sectors. Proposition 5 gives the condition of balanced growth with and without structural change. If the sectoral TFPs $\hat{\epsilon}_s$ are constant and the augmented rate of technological change is the same across sectors, the economy is on a balanced growth path with no structural change. If these two conditions are not met simultaneously, any balanced growth path will inevitably result in structural change.

Proposition 5 demonstrates that structural change is consistent with balanced growth in an economy where the sectoral production functions differ. Because (ii) is a knife-edge condition, it implies that conditional on balanced growth, the most likely scenario is for goods and services value-added shares to move in the opposite direction. Furthermore, the proposition demonstrates that an economy with balanced growth and structural change is incompatible with constant sectoral TFP growth. Indeed, growth is balanced under conditions (i) and (ii), but in this case, sectoral value-added are constant over time. If

(ii) is not met, there will be no balanced growth with constant sectoral TFPs growth[8]. In other words, balanced growth with structural change implies that the rates of sectoral TFP growth are not constant.

5. Empirical Analysis

In this section, I calibrate the parameters of the framework presented in section 1. I use the calibrated model to simulate the balanced growth path that matches the data. I find that this balanced growth path also induces structural change.

5.1. Data

I use the data constructed by Herrendorf et al. (2021). This data combines US industry data from WORLD KLEMS and the annual input-output tables from the BEA.

5.2. Calibration

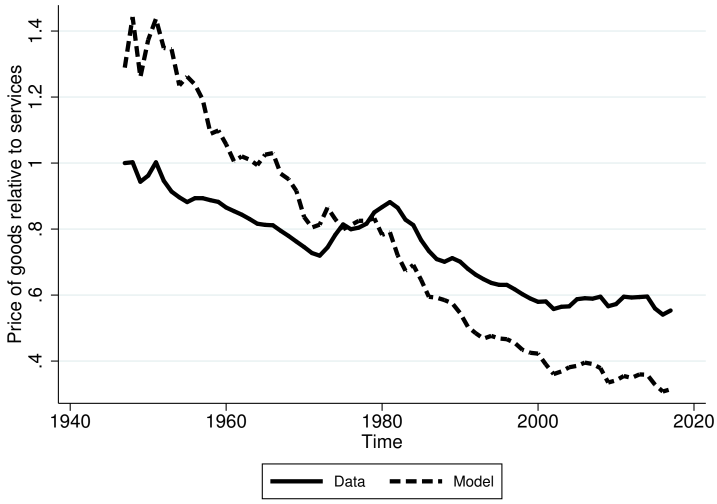
I calibrate two parameters internally: ρ and ψ. To calibrate these parameters, I consider the equation that gives the equality of relative prices and the marginal rate of transformation derived from the aggregate producer's optimization problem.

$$\frac{P_1}{P_2} = \left(\frac{\psi}{1-\psi}\right)^{\frac{1}{\rho}} \Delta^{\frac{1-\rho}{\rho}} \tag{22}$$

Let's recall that Δ is the relative value-added. The two parameters are calculated to minimize the distance between the series of the relative prices in the data and the series estimated with the relative value-added in equation (22). The value of ρ is -2.71, and the value of ψ equals 0.12. Figure 1 presents the actual and the predicted goods to services price ratio. The remaining parameters are set externally. As in Kaymak and Schott (2019), the parameter γ is set to 0.85. Profits account for 15% of the total income, which drives this value. The

[8]In light of proposition 5, assumptions 1 and 2 in AG rule out balanced growth in finite time.

Figure 1: Actual vs predicted relative prices



goods labor share and the services labor share are 0.61 and 0.54, respectively (Herrendorf et al. (2021)). It implies that α_1 is 0.24, and α_2 is 0.31. The interest rate is set to its long-term value of 4% (see Kaymak and Schott (2019)). Table 1 contains the summary of the calibrated parameters.

Table 1: Calibration summary

Description		Value	Target/Identification
α_1	Capital intensity in goods	0.24	labor share in goods of 61%
α_2	Capital intensity in services	0.31	labor share in services of 54%
γ	span of control	0.85	Profits share 15% (Kaymak and Schott (2019))
r	Interest rate	4%	Kaymak and Schott (2019)
ψ		0.12	
ρ		-2.71	

5.3. Simulation of the Balanced Growth Path

I calculate the TFP trajectory in goods and services to meet two conditions. First, the price ratio matches the corresponding values in the data. Second, the condition of balanced growth described in Proposition 1 is satisfied. The average growth rate of the real value-added is 3% between 1947 and 2017.

Figure 2: Aggregate value-added in the data and in the model

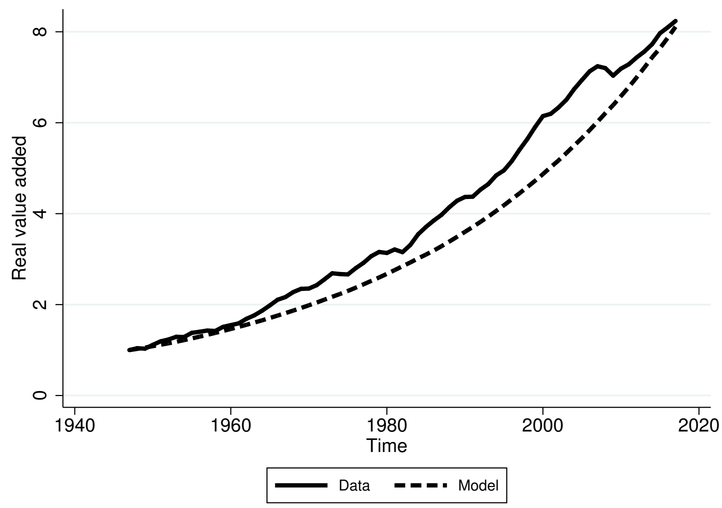
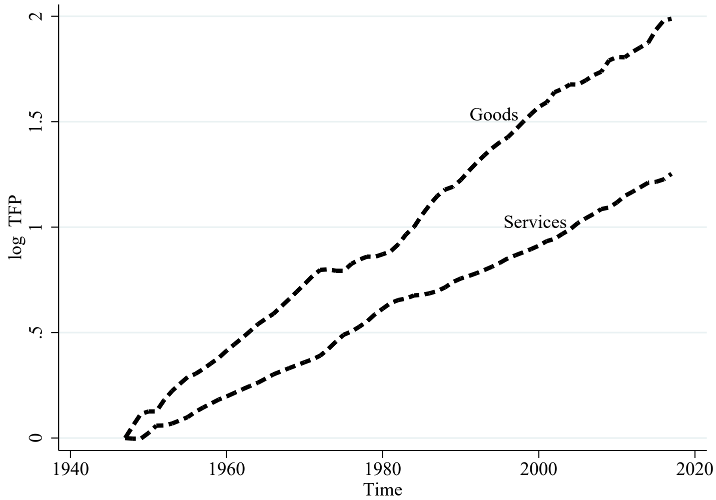


Figure 2 depicts the aggregate real value-added of the data and the model between 1947 and 2017. The solid line represents the aggregate value-added in the data, and the dashed line represents the model's aggregate value-added. The growth rate in the data is comparable to the balanced growth path generated by the model. Although several crises have contributed to fluctuations in GDP, long-term averages show that its growth rate has been relatively stable.

Figure 3 depicts changes in log TFP in goods and services that generates a constant growth rate of aggregated value-added of 3% such that changes in goods and services prices are consistent with the data. The graph illustrates two observations. First, the goods sector TFP grows faster than the services sector TFP. TFP growth is 3% in the goods sector and 2% in the services sector. The faster growth in TFP of the goods sector reduces the goods sector's marginal cost relative to the services sector. This has the effect of lowering the optimal output of services relative to goods. However, because goods and services are gross complements, the price of services soars sharply relative to the price of goods, causing the

Figure 3: TFP in goods and services sectors



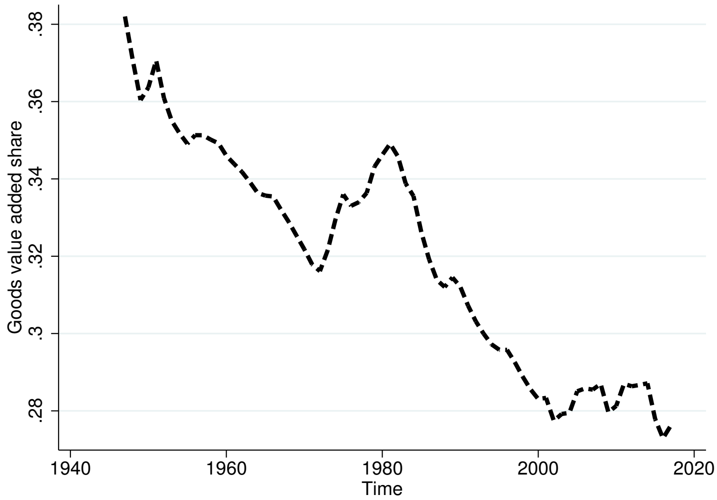
Note: The sectoral TFPs are estimated to such that aggregate value-added grows at a constant rate.

value-added of services to rise relative to the value-added of goods.

The second observation is that the rate of sectoral TFP growth is not constant, but quasi-constant. The two curves are, indeed, close to straight lines. Therefore, while constant TFP growth is incompatible with balanced growth in an economy undergoing structural change, quasi-constant TFP growth is. Indeed, the dynamics of the TFP depicted in Figure 3 implies a fall of the goods sectors value-added relative to the services value-added.

Figure 4 depicts the goods value-added share. The goods sector's value-added share has fallen from 38% in 1947 to 27% in 2017. This variation accounts for 50% of the data's actual change. Other channels, such as demand side factors, drive the remaining variation. While the aggregate growth rate remains constant, the value-added in the services sector grows faster than the value-added in the goods sector. As a result, along the balanced growth path, the value-added share decreases in the goods sector while increasing in the services sector. The simulated balanced growth, therefore, also features a structural change pattern.

Figure 4: Goods sector value-added share



Note: The goods sector value-added share is computed from the model along the balanced growth path.

6. Conclusion

This paper reconciles two stylized facts that characterize modern economic growth, balanced growth, and structural change, in a context where the factor intensities differ. Indeed, the rate of growth in the United States has been balanced since 1947. Furthermore, since 1947, the value-added share of goods has decreased while the value-added share of services has increased. As a result, the US economy has been undergoing structural change.

I extend the neoclassical growth model to two sectors, goods and services so that the factor intensities of the goods and the services differ. I characterize the dynamics of sectoral TFP in order to achieve balanced growth. In a two-sector framework with different factor intensities, the economy is on a balanced growth path if the average of sectoral TFP growth minus wage growth weighted by the aggregate labor share is constant.

In the two-sector framework with different factor intensities, structural change occurs via two channels. The first is the relative change of the sectoral TFP, and the second is the change in the relative cost of factors. The paper also demonstrates that balanced growth is compatible with structural change. The quantitative analysis reveals that the log of TFP dynamics is quasi-constant, TFP grows faster in the goods sector, the value-added decreases in the goods sector in relative terms, and aggregate growth is balanced. The model accounts for 50% of the actual structural change.

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Appendices

Appendix A. Balanced Growth

Appendix A.1. Proof of Equations (18) and (19)

$$
\begin{aligned}
& \psi^{\frac{1}{1-\rho}} P_1(t)^{-\frac{\rho}{1-\rho}} + (1-\psi)^{\frac{1}{1-\rho}} P_2(t)^{-\frac{\rho}{1-\rho}} = 1 \\
\implies & \left(\frac{\psi}{1-\psi}\right)^{\frac{1}{1-\rho}} \left(\frac{P_1(t)}{P_2(t)}\right)^{-\frac{\rho}{1-\rho}} + 1 = (1-\psi)^{-\frac{1}{1-\rho}} P_2(t)^{\frac{\rho}{1-\rho}} \\
\implies & -\frac{\rho}{1-\rho}\left(\hat{P}_1(t) - \hat{P}_2(t)\right)\left(1 - (1-\psi)^{\frac{1}{1-\rho}} P_2(t)^{-\frac{\rho}{1-\rho}}\right) = \frac{\rho}{1-\rho}\hat{P}_2(t) \\
\implies & -\left(\hat{P}_1(t) - \hat{P}_2(t)\right)\psi^{\frac{1}{1-\rho}} P_1(t)^{-\frac{\rho}{1-\rho}} = \hat{P}_2(t)
\end{aligned}
$$

From equation (11), we have $\psi^{\frac{1}{1-\rho}} P_1(t)^{-\frac{\rho}{1-\rho}} = S_1^Q$.

Equation (10) gives the expression of $\hat{P}_1(t) - \hat{P}_2(t)$ in terms of $\hat{w}$. $\hat{P}_1(t)$ is determined likewise.

Appendix A.2. Proof of Equation (16)

I use the labor market clearing condition, and I assume that the labor supply is constant.

$$
\begin{aligned}
& N_1(t) + N_2(t) = N(t) \\
\implies & S_1^N(t)\hat{N}_1(t) + S_2^N(t)\hat{N}_2(t) = 0 \\
\implies & S_1^N(t)\left(\hat{P}_1(t) + \hat{\epsilon}_1(t)\right) + S_2^N(t)\left(\hat{P}_2(t) + \hat{\epsilon}_2(t)\right) = \left(1 - \alpha_1 S_1^N(t) - \alpha_2 S_2^N(t)\right)\hat{w}(t)
\end{aligned}
$$

Next, I replace $\hat{P}_1(t)$ and $\hat{P}_2(t)$ by their respective expressions in equations (18) and (19).

Appendix A.3. Proof of Equation (15)

$$\begin{aligned}
& Q(t) = P_1(t)Q_1(t) + P_2(t)Q_2(t) \\
\Longrightarrow\ & \hat{Q}(t) = S_1^Q \left(\hat{P}_1(t) + \hat{Q}_1(t)\right) + S_2^Q \left(\hat{P}_2(t) + \hat{Q}_2(t)\right) \\
\Longrightarrow\ & \hat{Q}(t) = \frac{1}{1-\gamma}\left(S_1^Q \left(\hat{P}_1(t) + \hat{\epsilon}_1(t) - (\gamma-\alpha_1)\hat{w}(t)\right) + S_2^Q \left(\hat{P}_2(t) + \hat{\epsilon}_2(t) - (\gamma-\alpha_2)\hat{w}(t)\right)\right) \\
\Longrightarrow\ & (1-\gamma)\hat{Q}(t) = S_1^Q \left(\hat{P}_1(t) + \hat{\epsilon}_1(t) - (\gamma-\alpha_1)\hat{w}(t)\right) + S_2^Q \left(\hat{P}_2(t) + \hat{\epsilon}_2(t) - (\gamma-\alpha_2)\hat{w}(t)\right)
\end{aligned}$$

Next, I replace $\hat{P}_1$ and $\hat{P}_2$ by their respective expressions in equations (18) and (19). In addition, we have that $S_1^Q \hat{P}_1(t) + S_2^Q \hat{P}_2(t) = 0$. On the balanced growth path, $(1-\gamma)\hat{Q}(t)$ is constant.

Appendix A.4. Proof of Proposition 5

Under the condition of no structural change, the employment share and the value added share are constant over time. In this case, equation (16) implies that:

$$\hat{w} = x_1\hat{\epsilon}_1 + x_2\hat{\epsilon}_2$$

where x_1 and x_2 are constant numbers. The formal expression of x_1 and x_2 are given by the following:

$$x_1 = \frac{\rho(1-\gamma)S_1^N + (1-\rho)S_1^Q}{\rho(1-\gamma)\left(1 - S_1^N\alpha_1 - S_2^N\alpha_2\right) + (1-\rho)\left(1 - S_1^Q\alpha_1 - S_2^Q\alpha_2\right)}$$

$$x_2 = \frac{\rho(1-\gamma)S_2^N + (1-\rho)S_2^Q}{\rho(1-\gamma)\left(1 - S_1^N\alpha_1 - S_2^N\alpha_2\right) + (1-\rho)\left(1 - S_1^Q\alpha_1 - S_2^Q\alpha_2\right)}$$

Given the definition of structural change, as well as the expressions for the employment share in equation (12) and the value added share in equation (13) , structural change implies that

$\hat{\Delta}$ is not equal to zero. Therefore, balanced growth without structural change implies the following:

$$\begin{aligned}\hat{\Delta} = 0 &\implies \hat{\epsilon}_2 - \hat{\epsilon}_1 + (\alpha_2 - \alpha_1)\hat{w} = 0 \\ &\implies \hat{\epsilon}_2 - \hat{\epsilon}_1 + (\alpha_2 - \alpha_1)(x_1\hat{\epsilon}_1 + x_2\hat{\epsilon}) = 0 \\ &\implies \hat{\epsilon}_1 = \chi_0\hat{\epsilon}_2\end{aligned}$$

where χ_0 is defined by:

$$\chi_0 = \frac{(\alpha_2 - \alpha_1)x_2 + 1}{1 - (\alpha_2 - \alpha_1)x_1} = \frac{1 - \alpha_1}{1 - \alpha_2}$$

The wage rate can then be expressed in terms of the TFP growth in one sector.

$$\hat{w} = (\chi_0 x_1 + x_2)\,\hat{\epsilon}_2$$

The condition of balanced growth implies the following:

$$\left(S_1^Q\chi_0 + S_2^Q - \left(S_1^Q(\gamma - \alpha_1) + S_2^Q(\gamma - \alpha_2)\right)(\chi_0 x_1 + x_2)\right)\hat{\epsilon}_2 \text{ is constant}$$

Thus, since the term in parenthesis is constant, the condition implies that $\hat{\epsilon}_2$, and similarly $\hat{\epsilon}_1$, is constant.

Appendix B. Constant Return-to-Scale

Lemma 1. *Let (X, d) and (Y, d') be two metric spaces. A function $f : X \to Y$ is continuous at a point $a \in X$ if and only if whenever $\lim_{n\to\infty} a_n = a$ for a sequence $a_1, a_2, \ldots$ of points of X, $\lim_{n\to\infty} f(a_n) = f(a)$.*

Proof. see ch.2 theorem 5.4 of Mendelson (1990) □

Appendix B.1. Proof of Proposition 3

Since $\mathcal{M}$ and $\mathcal{F}_\gamma$ are continuous then $\mathcal{M} \circ \mathcal{F}_\gamma$ is continuous in γ and $\mathcal{M} \circ \mathcal{F}_\gamma = \Phi_\gamma$, $\forall \gamma < 1$. Let's now consider a given sequence $\{\gamma_n\}_{n\in\mathbb{N}}$ in [0,1) s.t. $\underset{n\to\infty}{lim}\, \gamma_n = 1$ (for example $\gamma_n = 1 - \frac{1}{n}, \forall n \in \mathbb{N}^*$). Using Lemma 1 we have the following:

$$\mathcal{M} \circ \mathcal{F}_1 = \mathcal{M} \circ \mathcal{F}_{\underset{n\to\infty}{lim} \gamma_n} = \underset{n\to\infty}{lim}\, \mathcal{M} \circ \mathcal{F}_{\gamma_n} = \underset{n\to\infty}{lim}\, \Phi_{\gamma_n} = \Phi_1$$

Appendix B.2. Proof of Proposition 4

The growth rate of the aggregate output is given in Appendix A.3. It is expressed as a function of γ as follows:

$$\Phi_\gamma = \frac{S_1^Q \hat{\epsilon}_1 + S_2^Q \hat{\epsilon}_2 - \Big(S_1^Q(\gamma - \alpha_1) + S_2^Q(\gamma - \alpha_2) \Big) \hat{w}}{1 - \gamma} \tag{B.1}$$

where $\hat{w}$ is given by equation (16). S_1^Q, S_2^Q, and $\hat{w}$ are all function of γ. In light of Proposition 3, the growth rate of the economy with CRS is equal to the limit of Φ_γ when γ tends to one.

In order to compute the limit, let us notice that the numerator and the denominator are all equal to 0, when $\gamma = 1$. In this case, I use L'Hôpital's rule which is stated in Lemma 2.

Lemma 2. *Let f and g be two differentiable functions on an open interval I. Let $x_0 \in I$, such that $f(x_0) = g(x_0) = 0$, and $g'(0) \neq 0$. Then*

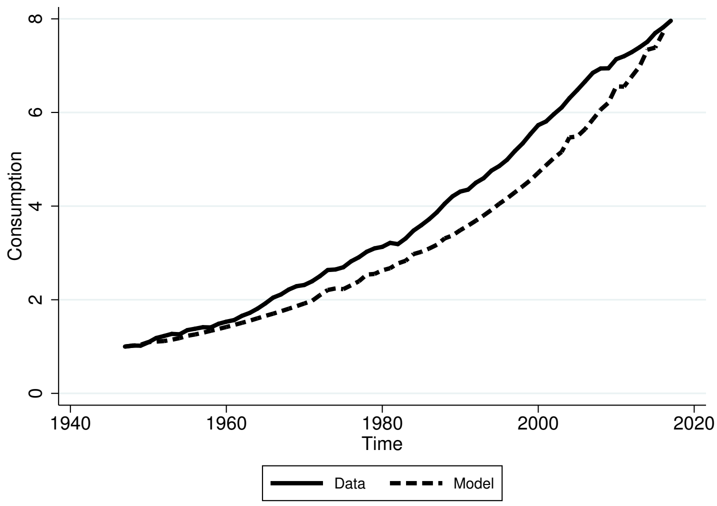
$$\underset{x\to x_0}{lim} \frac{f(x)}{g(x)} = \frac{f'(x_0)}{g'(x_0)}$$

In light of Lemma 2, we have the following

$$\Phi_1 = \underset{\gamma \to 1}{lim}\, \Phi_\gamma = -\frac{\partial\left(S_1^Q \hat{\epsilon}_1 + S_2^Q \hat{\epsilon}_2 - \left(S_1^Q(\gamma - \alpha_1) + S_2^Q(\gamma - \alpha_2)\right)\hat{w}\right)}{\partial \gamma}\Bigg|_{\gamma=1}$$

$$= \frac{\rho}{1-\rho}\left(S_1^Q - S_1^N\right)(\hat{\epsilon}_1 - \hat{\epsilon}_2) + \left[\frac{\rho}{1-\rho}\left(S_1^Q - S_1^N\right)(\alpha_1 - \alpha_2) + 1\right]\hat{w}$$

Aggregate consumption: Data vs Model



Note: The consumption in the data is taken from Herrendorf et al. (2021). The consumption in the model is calculated along the simulated balanced growth.