Balanced Growth and Structural Change

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Abstract

This paper reconciles two stylized facts that characterize modern economic growth, balanced growth and structural change, in a context where the factor intensities differ. I extend the neoclassical growth model to two sectors with different factor intensities, and I derive the dynamics of the sectoral TFPs that ensure aggregate balanced growth. I derive the condition on the TFP growth such that balanced growth is consistent with structural change. The condition of balanced growth in a two-sector model with different factor intensities is that the aggregate TFP growth minus the wage growth weighted by the aggregate labor share is constant. In this framework, structural change occurs through two channels. The first is the change in the sectoral TFP ratio and the second is the change in the relative cost of factors. The empirical analysis confirms that the model replicates the stylized facts aforementioned.

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Introduction

This paper reconciles two stylized facts that characterize modern economic growth, balanced growth and structural change, in a context where the factor intensities differ. I extend the neoclassical growth model to two sectors with different factor intensities, and I derive the dynamics of the sectoral TFPs that ensure aggregate balanced growth. I derive the condition on the TFP growth such that balanced growth is consistent with structural change.

Balanced growth is characterized by a constant growth rate of the aggregate consumption and the aggregate output. The condition of balanced growth in a two-sector model with different factor intensities is that the aggregate TFP growth minus the wage growth weighted by the aggregate labor share is constant. The aggregate TFP growth rate is calculated as the weighted mean of the sectoral TFP growth rates, where the weights represent the sectoral value added shares. The aggregate labor share is the weighted mean of the sectoral labor share, where the weights are also the sectoral value added shares. The two-sector framework's balanced growth differs from neoclassical balanced growth in two ways. First, the sectoral TFP growth rates are not constant and vary according to sectoral TFP levels. Second, the growth of wages is not necessarily zero.

Structural change implies that the value added share and the employment share increase in one sector while decreasing in the other. In this framework, structural change occurs through two channels. The first is the change in the sectoral TFP ratio and the second is the change in the relative cost of factors. In the case where goods and services are gross complement, if the relative cost of labor rises, then the value added share of the labor-intensive sector increases, and vice versa. The opposite occurs if the sectors are gross substitute.

I demonstrate that balanced growth is compatible with structural change. Balanced growth without structural change is characterized by a knife-edge condition. Under this condition, the TFP growth rates in both sectors are constant, and the ratio of their growth rates is equal to the ratio of the factor intensities. Any other balanced growth path that does not meet this condition results in a structural change pattern.

I develop a model with a decreasing return-to-scale production function. The results for a decreasing return-to-scale are then extended to a constant return-to-scale case.

I calibrate the model to the US economy and generate the TFP in such a way that the ratio of the TFP in the goods sector and the services sector matches the data, as well as the aggregate value added growth is constant. I find that the relative prices in the model along the balanced growth path are similar to the data. The goods sector value added and employment shares are decreasing, while the services sector value added and employment shares are increasing. Thus, the model replicates both structural change and balanced growth as in the data.

The first stylized fact that the paper elicits is balanced growth. Kaldor (1961) showed that the output per worker and the capital per worker have steadily increased, while the capital-output ratio and the real return on capital have remained relatively constant. For nearly 150 years, real GDP per capita in the United States has grown at a remarkably steady average rate of around 2% per year, while the ratio of physical capital to output has remained nearly constant (Jones (2016)).

The second fact is structural change. Simon Kuznets in his Nobel prize speech has listed structural change as one of the six characteristics of modern economic growth. In 1870, agriculture employed 40% of all Americans. Agriculture accounted for only 4% of employment 100 years later (Kongsamut et al. (2001)). Herrendorf et al. (2021) documented that, while the value added share and the employment share of the goods sector have decreased since 1947, the value added share and the employment share of the services sector have increased.

At first glance, it appears that the two stylized facts, secular balanced growth and structural change, are contradictory. Indeed, on the one hand, balanced growth reflects the fact that economies have been on a constant growth path for a long period of time. One property of balanced growth models is that the proportion of labor and capital allocated to different industries remains constant over time (Kongsamut et al. (2001)). On the other hand, structural change shows how the goods sector has shrunk over time and the services sector took over (Herrendorf et al. (2021)).

This paper is related to several works, including Kongsamut et al. (2001), Ngai and Pissarides (2007), Herrendorf et al. (2021). The first two papers derive the constraints on household preferences that allow balanced growth to be consistent with structural change. The third paper derives the dynamics of the TFP such that the aggregate value added growth is balanced and finds that balanced growth condition is also consistent with structural change. The primary contribution of this paper to this line of the literature is to examine two-sector economies with different factor intensities; derive the dynamics of the TFP in this framework for aggregate balanced growth, as well as the condition under which balanced growth is consistent with structural change.

The data motivates the consideration of a two-sector economy with different factor intensities. Indeed, the postwar period saw a decrease in the labor share of the goods sector and an increase in the labor share of the services sector[1].

The paper is also related to Acemoglu and Guerrieri (2008) and Alvarez-Cuadrado et al. (2017). They considered a two-sector model with different factor intensities and derive the asymptotic balanced growth. This paper shows the existence of balanced growth in finite time.

The paper is organized as follows. In the first section, I present a two-sector economy framework with different factor intensities. I describe the equilibrium factor behavior in the second section. I characterize the TFP dynamic that generates balanced growth in the third section. The fourth section examines the condition under which balanced growth is compatible with structural change. The fifth section extends the result with decreasing return-to-scale to the case with constant return-to-scale. Finally, I simulate the balanced growth path in the last section and find that the behavior of the sectoral value added shares along the balanced growth path is consistent with the data.

[1]See the data constructed by Herrendorf et al. (2021)

1. Two-Sector Economy Framework

In this section, I develop a framework of two-sector economy. The framework is a neoclassical general equilibrium growth model with two sectors, goods and services. The factor intensities in the goods sector and the services sector differ. I use a production function with a decreasing return-to-scale. The primary reason for this is that the decreasing return-to-scale framework clearly defines the quantities of factors and the value added. However, in section 5, I extend the decreasing return-to-scale framework's results to a constant return-to-scale case.

1.1. Preferences

The framework features a representative household. The lifetime consumption yields a utility represented by the following function:

$$U = \int_0^\infty \mathrm{e}^{-\varrho t} \frac{C(t)^{1-\sigma}}{1-\sigma} \mathrm{d}t \tag{1}$$

where $C(t)$ is the consumption of period t. The total income is the sum of the capital income $r(t)K(t)$, the labor income $w(t)N(t)$ and the firms profit $\Pi(t)$. The total expenditure is the sum of the consumption expenditure $P(t)C(t)$ and the capital expenditure $P(t)\dot{K}(t)$. $P(t)$ is normalized to one. Formally, the budget constraint is:

$$\dot{K}(t) + C(t) = r(t)K(t) + w(t)N(t) + \Pi(t) \tag{2}$$

1.2. Firms

The economy is populated by three firms. The first firm produces goods denoted by Q_1; the second firm produces services denoted by Q_2. The third firm is an aggregate producer and combines goods and services to produce the gross output denoted by Q. The production of goods and services uses capital and labor according to the following function.

$$Q_s(t) = \epsilon_s(t) K_s(t)^{\alpha_s} N_s(t)^{\gamma-\alpha_s}, \qquad s \in \{1,2\} \tag{3}$$

where ϵ_s and α_s are the total factor productivity (TFP), and the capital intensity of firm s, respectively. K_s and N_s represent the amount of capital and labor used by the firm s, respectively.

The production function is characterized by a decreasing return-to-scale, in other words, $\alpha_s < \gamma < 1$. Let $r(t)$ and $w(t)$ be the interest rate and the wage rate at period t, respectively. The firms are price takers, and their respective problems are formalized as follows:

$$\max_{N_s(t), K_s(t)} P_s(t)\epsilon_s(t) K_s(t)^{\alpha_s} N_s(t)^{\gamma-\alpha_s} - w(t)N_s(t) - r(t)K_s(t), \qquad s \in \{1,2\} \tag{4}$$

where P_s denotes the price of the output of the firm s.

The aggregate producer uses Q_1, and Q_2, the value added in the goods and the services sectors respectively, to produce the aggregate output Q. Her production function is the following:

$$Q(t) = \left(\psi Q_1(t)^\rho + (1-\psi) Q_2(t)^\rho\right)^{\frac{1}{\rho}} \tag{5}$$

ψ is the weight of goods and $1-\psi$ is the weight of services in the production of the aggregate output. ρ is the substitution parameter such that the elasticity of substitution between goods and services equals $\frac{1}{1-\rho}$. If $\rho < 0$, then goods and services are gross complement, otherwise they are gross substitute. The aggregate producer maximizes her profit by taking the prices of the inputs and the output as given.

$$\max_{Q_1(t), Q_2(t)} P(t)\left(\psi Q_1(t)^\rho + (1-\psi) Q_2(t)^\rho\right)^{\frac{1}{\rho}} - P_1(t)Q_1(t) - P_2(t)Q_2(t) \tag{6}$$

1.3. Equilibrium

Let $K(0)$ be the initial stock of capital. A competitive equilibrium is a set of prices $\{P_1(t), P_2(t), r(t), w(t)\}$, a total consumption $C(t)$, a stock of capital $K(t)$, the sectoral value added $Y_s(t)$, and the aggregate value added $Y(t)$, such that:

(i) Given the price of the gross output, $C(t)$ and $\dot{K}(t), t \in \mathbb{N}$ maximize the household lifetime utility.

(ii) Given the prices of the factors of production, $N_s(t)$, and $K_s(t)$ maximize the firm s profit; $Q_1(t)$, and $Q_2(t)$ maximize the aggregate producer's profit.

(iii) The capital market clears

$$K_1(t) + K_2(t) = K(t) \tag{7}$$

(iv) The labor market clears

$$N_1(t) + N_2(t) = N(t) \tag{8}$$

(v) The aggregate output market clears

$$\dot{K}(t) + C(t) = Q(t) \tag{9}$$

2. Model Analysis

In this section, I calculate the factor demands by sector. I compute the optimal value added as well as the prices for each sector. Finally, I compute the value added and employment shares of the goods sector and the services sector.

Factor demands are determined by the following equations:

$$w(t)N_s(t) = (\gamma - \alpha_s)P_s(t)^{\frac{1}{1-\gamma}}\epsilon_s(t)^{\frac{1}{1-\gamma}}\left(\frac{\alpha_s}{r(t)}\right)^{\frac{\alpha_s}{1-\gamma}}\left(\frac{\gamma-\alpha_s}{w(t)}\right)^{\frac{\gamma-\alpha_s}{1-\gamma}}$$

$$r(t)K_s(t) = \alpha_s P_s(t)^{\frac{1}{1-\gamma}} \epsilon_s(t)^{\frac{1}{1-\gamma}} \left(\frac{\alpha_s}{r(t)}\right)^{\frac{\alpha_s}{1-\gamma}} \left(\frac{\gamma-\alpha_s}{w(t)}\right)^{\frac{\gamma-\alpha_s}{1-\gamma}}$$

The optimal real value added in the goods sector and the services sector is defined by the following equation:

$$Q_s(t) = P_s(t)^{\frac{\gamma}{1-\gamma}} \epsilon_s(t)^{\frac{1}{1-\gamma}} \left(\frac{\alpha_s}{r(t)}\right)^{\frac{\alpha_s}{1-\gamma}} \left(\frac{\gamma-\alpha_s}{w(t)}\right)^{\frac{\gamma-\alpha_s}{1-\gamma}}$$

where $s = 1, 2$.

The aggregate output price $P(t)$ is normalized to one; therefore, we have the following equation:

$$P(t) = \left[\psi^{\frac{1}{1-\rho}} P_1(t)^{-\frac{\rho}{1-\rho}} + (1-\psi)^{\frac{1}{1-\rho}} P_2(t)^{-\frac{\rho}{1-\rho}}\right]^{-\frac{1-\rho}{\rho}} = 1$$

The first-order condition of the aggregate producer's problem is that the marginal rate of transformation equals the relative prices. I compute the relative prices by combining this optimal condition with the sectoral value added derived above.

$$\frac{P_1(t)}{P_2(t)} = \left(\frac{\psi}{1-\psi}\right)^{\frac{1-\gamma}{1-\gamma\rho}} \left[\theta \frac{\epsilon_1(t)}{\epsilon_2(t)} \left(\frac{w(t)}{r(t)}\right)^{\alpha_1-\alpha_2}\right]^{-\frac{1-\rho}{1-\gamma\rho}} \tag{10}$$

where $\theta = \frac{\alpha_1^{\alpha_1}(\gamma-\alpha_1)^{\gamma-\alpha_1}}{\alpha_2^{\alpha_2}(\gamma-\alpha_2)^{\gamma-\alpha_2}}$. When the sectors have the same factor intensities, only the TFP ratio determines the relative prices. When the intensities of the factors of production differ, the ratio of the cost of the factors of production also affects the relative prices. The relative price of the labor-intensive sector rises as the relative cost of labor increases.

Next, I determine the value added share and the employment share of the goods sector and the services sector. Let us denote by $\Delta = \frac{P_2 Q_2}{P_1 Q_1}$, the nominal value added of the services sector relative to the goods sector. This ratio depends on the ratio of the TFPs and the relative cost of factors.

$$\Delta(t) = \left(\frac{1-\psi}{\psi}\right)^{\frac{1}{1-\gamma\rho}} \left(\frac{1}{\theta}\frac{\epsilon_2(t)}{\epsilon_1(t)}\left(\frac{w(t)}{r(t)}\right)^{\alpha_2-\alpha_1}\right)^{\frac{\rho}{1-\gamma\rho}}$$

The prices of goods and services are given by:

$$P_1(t) = \psi^{\frac{1}{\rho}} \left(\Delta(t)+1\right)^{\frac{1-\rho}{\rho}}; \qquad P_2(t) = (1-\psi)^{\frac{1}{\rho}} \left(\Delta(t)^{-1}+1\right)^{\frac{1-\rho}{\rho}} \tag{11}$$

Because the TFP reduces the marginal cost of production, an increase in the TFP of a given sector lowers its relative price.

The employment share of the sector s is the amount of labor used in sector s relative to the total labor $\frac{N_s}{N_1+N_2}$. Let S_1^N and S_2^N denote the employment share of the goods sector and the services sector, respectively.

$$S_1^N(t) = \frac{1}{1+\frac{\gamma-\alpha_2}{\gamma-\alpha_1}\Delta(t)}; \qquad S_2^N(t) = \frac{1}{1+\frac{\gamma-\alpha_1}{\gamma-\alpha_2}\Delta(t)^{-1}} \tag{12}$$

The value added share of the sector s is the proportion of the value added of the sector s in the total value added $\frac{P_sQ_s}{P_1Q_1+P_2Q_2}$. Let S_1^Q and S_2^Q denote the value added share of the goods sector and the services sector, respectively.

$$S_1^Q(t) = \frac{1}{1+\Delta(t)}; \qquad S_2^Q(t) = \frac{1}{1+\Delta(t)^{-1}} \tag{13}$$

If goods and services are complement, then a relative increase in the TFP of a given sector reduces its employment and value added shares by lowering its relative price. When the sectors are substitute, the opposite occurs.

3. Balanced Growth

In this section, I derive the condition of balanced growth. According to neoclassical theory, balanced growth occurs when all factors grow at the same rate. However, such a definition is too narrow and can not be applied to an economy with structural change. I then use the definition of Kongsamut et al. (2001) stated as follows:

Definition 1. *A balanced growth path is a trajectory along which the real interest rate is constant.*

According to this definition, aggregate consumption grows at a constant rate along the balanced growth path. However, the rate of growth in sectoral output does not have to be constant along the balanced growth path. The optimal path of the consumption is determined by the Euler equation.

$$\hat{C}(t) = \frac{1}{\sigma}(r(t) - \varrho) \tag{14}$$

The *hat* sign represents the growth rate. Regarding the Euler equation, a constant interest rate implies that the total consumption grows at a constant rate. Since the accumulation of capital $\dot{K}$ is proportional to the consumption C along the balanced growth path (see Moll et al. (2019)), the necessary and sufficient condition of balanced growth is a constant growth of the aggregate output, regarding equation (9).

Hereafter, I drop the time symbol to simplify the notation. The next proposition states the necessary and sufficient condition of balanced growth of an economy with two sectors and different factor intensities.

Proposition 1. *For any initial condition $K(0) > 0$, the equilibrium allocation is on a balanced growth path for all $t > 0$ if and only if the sectoral TFP sequences satisfy the* *condition that*

$$S_1^Q\hat{\epsilon}_1 + S_2^Q\hat{\epsilon}_2 - \Big(S_1^Q(\gamma-\alpha_1) + S_2^Q(\gamma-\alpha_2)\Big)\,\hat{w} \text{ is constant} \tag{15}$$

where

$$\hat{w} = \frac{\rho(1-\gamma)\left(S_1^N\hat{\epsilon}_1 + S_2^N\hat{\epsilon}_2\right) + (1-\rho)\Big(S_1^Q\hat{\epsilon}_1 + S_2^Q\hat{\epsilon}_2\Big)}{\rho(1-\gamma)\left(1 - S_1^N\alpha_1 - S_2^N\alpha_2\right) + (1-\rho)\Big(1 - S_1^Q\alpha_1 - S_2^Q\alpha_2\Big)} \tag{16}$$

Proof. See Appendix A □

Proposition 1 states the condition of balanced growth in a two-sector growth model with different factor intensities in equation (15), as well as the wage growth rate in equation (16). Balanced growth implies that the aggregate TFP growth minus the wage growth weighted by the aggregate labor share is constant. In addition, wage growth is the weighted sum of the sectoral TFP growth rates. The aggregate TFP growth rate is calculated as the weighted mean of the sectoral TFP growth rates, where the weights represent the sectoral value added share. The aggregate labor share is the weighted mean of the sectoral labor share, where the weights are also the sectoral value added share.

The balanced growth path condition in this framework differs from the neoclassical balanced growth path condition in two ways. First, the sectoral TFP growth rates are not constant and vary according to sectoral TFP levels. Indeed the value added share and the employment share are functions of the sectoral TFP levels. Second, the cost of labor is not necessarily constant along the balanced growth path. However, the condition includes balanced growth paths with constant wages. Corollary 1 characterizes the balanced growth path with a constant wage rate.

Corollary 1. *For any initial condition $K(0) > 0$, the equilibrium allocation is on a balanced growth path with a constant wage rate for all $t > 0$ if and only if the sectoral TFP sequences* *satisfy the condition that*

$$\begin{cases} \hat{e}_1 = \frac{\kappa\left(\frac{(1-\gamma)\rho}{1-\gamma\rho}S_1^N + \frac{1-\rho}{1-\gamma\rho}S_1^Q - 1\right)}{S_1^Q - S_1^N} \\ \hat{e}_2 = \frac{\kappa\left(\frac{(1-\gamma)\rho}{1-\gamma\rho}S_1^N + \frac{1-\rho}{1-\gamma\rho}S_1^Q\right)}{S_1^Q - S_1^N} \end{cases} \tag{17}$$

where κ is a real number s.t. $\rho\kappa < 0$.

If goods and services are complement, balanced growth with constant wage rate results in faster TFP growth in the labor-intensive sector. If the sectors are substitute, then the balanced growth path implies that the capital-intensive sector's TFP grows faster.

The next proposition shows the sectoral dynamics of prices, factors of production, and the value added.

Proposition 2. *Along the balanced growth path, the sectoral aggregate growth rates are determined by:*

$$\hat{P}_1 = \frac{1-\rho}{1-\gamma\rho}(1-\psi)^{\frac{1}{1-\rho}}\left(\hat{\epsilon}_2 - \hat{\epsilon}_1 + (\alpha_2 - \alpha_1)\hat{w}\right)P_2^{-\frac{\rho}{1-\rho}} \tag{18}$$

$$\hat{P}_2 = \frac{1-\rho}{1-\gamma\rho}\psi^{\frac{1}{1-\rho}}\left(\hat{\epsilon}_1 - \hat{\epsilon}_2 + (\alpha_1 - \alpha_2)\hat{w}\right)P_1^{-\frac{\rho}{1-\rho}} \tag{19}$$

$$\hat{w} + \hat{N}_s = \hat{K}_s = \hat{P}_s + \hat{Q}_s = \frac{1}{1-\gamma}\left(\hat{P}_s + \hat{\epsilon}_s - (\gamma - \alpha_s)\hat{w}\right) \tag{20}$$

Proof. See Appendix A □

Equations (18) and (19) represent the growth rates of prices of goods and services, respectively. Prices rise more quickly in the slowest TFP sector. The increase in wages accelerates price increases in the labor-intensive sector while slowing price increases in the capital-intensive sector.

Equation (20) shows that the growth rates of the sectoral labor income, the capital income and the value added are identical and depend on the growth rate of the sectoral prices, TFPs, and the change in the wage rate. While the rise in the TFP and the output prices increases the income and the value added, the rise in wages has the opposite effect.

4. Structural Change

In this section, I investigate structural change along the balanced growth path. The definition of structural change is given below.

Definition 2. *Structural change implies that the value added share or similarly the employment share increases in one sector and decreases in another sector.*

The US economy is experiencing structural change, as it shifts from good-intensive to service-intensive mode. Indeed, the value added share and the employment share increase in the services sector and decrease in the goods sector (Herrendorf et al. (2021)).

If the sectors have identical factor intensities, structural change is caused solely by changes in the TFP ratio. With different factor intensities, however, structural change occurs not only through the TFP ratio, but also through changes in the relative cost of factors (see equations (12), and (13)). In the case where goods and services are gross complement, if the relative cost of labor rises, then the value added share of the labor-intensive sector increases, and vice versa. The opposite occurs if the sectors are gross substitute.

Structural change is consistent with balanced growth. The following proposition describes the condition of balanced growth with and without structural change.

Proposition 3. *Let $K(0)$ be the initial stock of capital. The equilibrium allocation is on a balanced growth path without structural change for all $t > 0$ if and only if the sectoral TFP sequences satisfy the condition that $\hat{\epsilon}_1$ and $\hat{\epsilon}_2$ are both constant, and $\frac{\hat{\epsilon}_1}{1-\alpha 1} = \frac{\hat{\epsilon}_2}{1-\alpha 2}$. Any other balanced growth path features necessarily a structural change pattern.*

Proof. See Appendix A $\square$

Proposition 3 gives the condition of balanced growth with and without structural change. If $\hat{\epsilon}_1$, and $\hat{\epsilon}_2$ are both constant, and if $\frac{\hat{\epsilon}_1}{1-\alpha 1} = \frac{\hat{\epsilon}_2}{1-\alpha 2}$, then the economy is on a balanced growth path without structural change. If these two conditions are not met, any balanced growth path will inevitably result in structural change.

5. Balanced Growth with Constant Return-to-Scale

The condition of balanced growth stated in Proposition 1 is derived with decreasing return-to-scale (DRS) in the goods sector and the services sector. In this section, I derive the condition of balanced growth with constant return-to-scale (CRS). To that end, I use Lemma 1 to extend DRS to CRS. According to Lemma 1, the limit and function symbols commute in a metric space for a continuous mapping.

Let's recall that γ is the span of control of the production function of goods and services. $\gamma < 1$ represents DRS and $\gamma = 1$ represents CRS. Let's denote by $\mathcal{F}_\gamma$, a framework characterized by a set of continuous functions in a metric space (X, d). Let's also denote by Φ_γ the outcome of the framework $\mathcal{F}_\gamma$. The outcome of the framework represents the set of the framework's analytical results. If the limit Φ_1 of Φ_γ exists when the span of control tends to unity, then Φ_1 is the analytical solution of the framework with CRS. Proposition 4 states this result more formally.

Proposition 4. *Let γ be a real number smaller than one. Let $\mathcal{F}_\gamma$ be a framework characterized by a set of continuous functions in a metric space (X, d), where the production function has the return-to-scale γ. Let Φ_γ be the set of analytical results of $\mathcal{F}_\gamma$, $\forall \gamma < 1$ s.t. the mapping $\mathcal{M} : \mathcal{F}_\gamma \longmapsto \Phi_\gamma$ is continuous.*
If $\Phi_1 = \lim\limits_{\gamma \to 1} \Phi_\gamma$ exists, then the analytical results of $\mathcal{F}_1$, the variant of $\mathcal{F}_\gamma$ where the production function has a constant return to scale, is Φ_1.

Proof. See Appendix B □

The outcome of a model with CRS is the limit of the outcome of a model with DRS when $\gamma \to 1$. The mapping $\mathcal{M}$ is continuous if all of the transformations used to get to the result are continuous. In light of Proposition 4, I derive the condition of balanced growth in an economy with CRS by using the results of section 3. In this case, the outcome Φ_γ represents the growth rate of the aggregate output. With DRS, $\Phi_\gamma = \frac{S_1^Q \hat{\epsilon}_1 + S_2^Q \hat{\epsilon}_2 - \left(S_1^Q(\gamma - \alpha_1) + S_2^Q(\gamma - \alpha_2)\right)\hat{w}}{1-\gamma}$, with $\hat{w}$ given in equation (16).

According to Proposition 4, if Φ_1 exists, then Φ_1 is the growth rate of the aggregate output with CRS. Appendix B shows that Φ_1 exists; therefore, the economy with CRS is on the balanced growth path if Φ_1 is constant. Proposition 5 states it formally.

Proposition 5. *Let us assume that $\gamma = 1$. For any initial condition $K(0) > 0$, the equilibrium allocation is on a balanced growth path for all $t > 0$ if and only if the sectoral TFP sequences satisfy the condition that*

$$S_1^N \hat{\epsilon}_1 + S_2^N \hat{\epsilon}_2 - \left(S_1^N (1 - \alpha_1) + S_2^N (1 - \alpha_2) + \frac{1-\rho}{\rho} \right) \hat{w} \text{ is constant} \tag{21}$$

Proof. See Appendix B □

Balanced growth, in the CRS case, implies that the aggregate TFP growth minus the wage growth weighted by the aggregate labor share is constant. In this case, the aggregate TFP growth is the weighted mean of sectoral TFP growth rates, where the weights are the employment shares. The aggregate labor share is the weighted mean of the sectoral labor shares adjusted for the elasticity of substitution between goods and services. The employment share and the wage growth rate are given in equation (12) and (16) respectively, where γ is equal to one.

6. Empirical Analysis

In this section, I calibrate the parameters of the framework presented in section 1. I use the calibrated model to simulate the balanced growth path that matches the data. I find that this balanced growth path also induces structural change.

6.1. Data

I use the data constructed by Herrendorf et al. (2021). This data combines US industry data from WORLD KLEMS and the annual input-output tables from the BEA. The data gives the value added, the labor income, and the TFP of goods and services from 1947 to 2017.

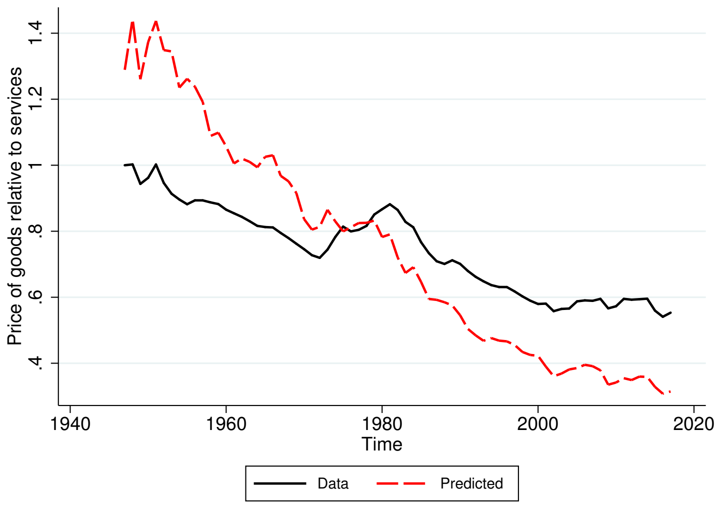
6.2. Calibration

I calibrate two parameters internally: ρ and ψ. To calibrate these parameters, I consider the equation that gives the equality of relative prices and the marginal rate of transformation derived from the aggregate producer's optimization problem.

$$\frac{P_1}{P_2} = \left(\frac{\psi}{1-\psi}\right)^{\frac{1}{\rho}} \Delta^{\frac{1-\rho}{\rho}} \tag{22}$$

Let's recall that Δ is the relative value added. The two parameters are calculated to minimize the distance between the series of the relative prices in the data and the series estimated with the relative value added in equation (22). The value of ρ is -2.71, and the value of ψ equals 0.12. Figure 1 presents the actual and the predicted goods to services price ratio. The

Figure 1: Actual vs predicted relative prices



remaining parameters are set externally. As in Kaymak and Schott (2019), the parameter γ is set to 0.85. Profits account for 15% of the total income, which drives this value. The goods labor share and the services labor share are 0.61 and 0.54, respectively (Herrendorf et al. (2021)). It implies that α_1 is 0.24, and α_2 is 0.31. The interest rate is set to its long-term value of 4% (see Kaymak and Schott (2019)). Table 1 contains the summary of the calibrated parameters.

Table 1: Calibration summary

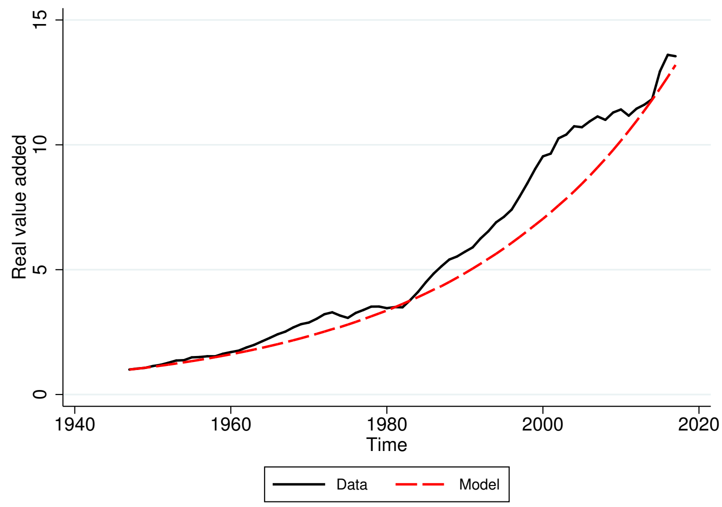
Description		Value	Target/Identification
α_1	Capital intensity in goods	0.24	labor share in goods of 61%
α_2	Capital intensity in services	0.31	labor share in services of 54%
γ	span of control	0.85	Profits share 15% (Kaymak and Schott (2019))
r	Interest rate	4%	Kaymak and Schott (2019)
ψ		0.12	
ρ		-2.71	

6.3. Simulation of the Balanced Growth Path

I calculate the TFP trajectory in goods and services to meet two conditions. First, the TFP ratio matches the corresponding values in the data. Second, the condition of balanced growth described in Proposition 1 is satisfied. The average growth rate of the real value added is 3.73% between 1947 and 2017. Figure 2 depicts the aggregate real value added of the data and the model between 1947 and 2017. The black solid line represents the aggregate value added of the data, and the red dashed line represents the model's aggregate value added. The growth rate in the data is comparable to the balanced growth path generated by the model. Although several crises have contributed to fluctuations in GDP, long-term averages show that its growth rate has been relatively stable.

Figure 3 depicts changes in the relative prices. The black line represents the relative prices as calculated from the data. The red line represents the relative prices along the balanced growth path simulated from the model. The model replicates the data pattern with a lower slope. Along the balanced growth path, the price of goods decreases relative to services. The decrease in the relative price of the goods sector results from the relative

Figure 2: Aggregate real value added in the data and in the model

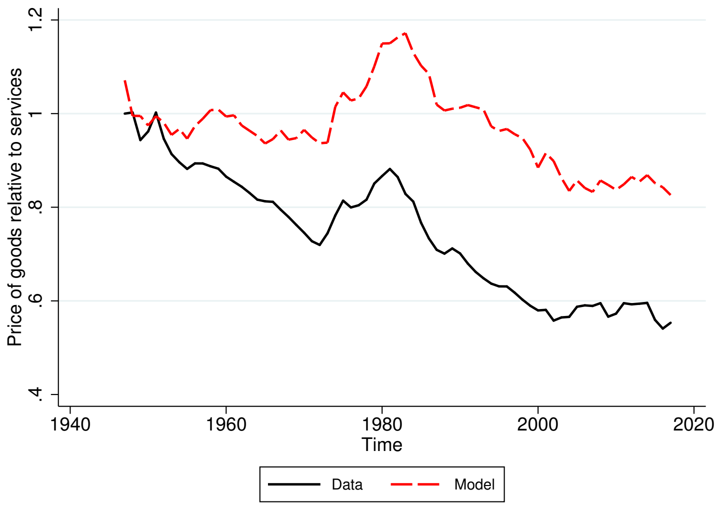


increase in its TFP. The gradient of the change in the price ratio is smaller in the model because the model assumes that labor supply remains constant over time. This causes a rapid increase in the relative cost of labor in the model compared to the data.

In figure 4, the goods value added share is represented by the dashed line, and the goods employment share is represented by the solid line. These changes occur along the balanced growth path. While the aggregate growth rate remains constant, the value added in the services sector grows faster than the value added in the goods sector. As a result, along the balanced growth path, the value added share and the employment share decrease in the goods sector while increasing in the services sector. The simulated balanced growth, therefore, also features structural change pattern. Because labor intensity is higher in the goods sector than in the services sector, the value added share is lower than the employment share in the goods sector.

Although the total output growth is balanced, the TFP in the goods sector has increased faster than the TFP in the services sector. The relative price of the services sector, which has the slowest TFP growth, rises. Because the sectors are gross complement, the relative price of services rises rapidly. As a result, the value added share of services rises while the

Figure 3: Relative prices in the data and in the model



value added share of goods falls.

7. Conclusion

This paper reconciles two stylized facts that characterize modern economic growth, balanced growth, and structural change, in a context where the factor intensities differ. Indeed, the rate of growth in the United States has been balanced since 1947, with the growth rate of GDP remaining roughly constant at 3.73% per year on average. Furthermore, since 1947, the value added share of goods has decreased while the value added share of services has increased. As a result, the US economy has been undergoing structural change.

I extend the neoclassical growth model to two sectors, goods and services so that the factor intensities of the goods and the services differ. I characterize the dynamics of sectoral TFP in order to achieve balanced growth. In a two-sector framework with different factor intensities, the economy is on a balanced growth path if the average of sectoral TFP growth minus wage growth weighted by the aggregate labor share is constant. The two-sector framework with different factor intensities balanced growth differs from neoclassical balanced growth in two ways. First, the rate of variation in sectoral TFPs is not constant and depends

Figure 4: Employment share and value added share

on the levels of the sectoral TFPs. Second, the growth of wages does not have to be zero.

In the two-sector framework with different factor intensities, structural change occurs via two channels. The first is the relative change of the sectoral TFP, and the second is the change in the relative cost of factors. Let us consider the situation in which the relative cost of labor rises. If the sectors are complement, the value added share of the labor-intensive sector increases. The opposite occurs if the sectors are substitute. The paper also demonstrates that balanced growth is compatible with structural change. Only under a knife-edge condition, balanced growth occurs without structural change.

The framework is built with DRS. The primary reason is that the DRS framework clearly defines the quantities of factors and output. I also show how DRS framework is extended to CRS.

I calibrate the model to the US economy. I find that the simulated balanced growth also features structural change. Along the balanced growth path, the value added and employment shares are decreasing in the goods sector while increasing in the services sector. Therefore, the model replicates the fact that the US aggregate value added growth is balanced and that the economy shifts from a good-intensive to a service-intensive mode. The model also replicates the mechanism underlying this fact, which is the faster growth of TFP in goods relative to services, as well as the rapid increase in the price of services relative to goods.

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Appendices

Appendix A. Balanced Growth

Appendix A.1. Proof of Equations (18) and (19)

$$
\begin{aligned}
& \psi^{\frac{1}{1-\rho}} P_1(t)^{-\frac{\rho}{1-\rho}} + (1-\psi)^{\frac{1}{1-\rho}} P_2(t)^{-\frac{\rho}{1-\rho}} = 1 \\
\Longrightarrow & \left(\frac{\psi}{1-\psi}\right)^{\frac{1}{1-\rho}} \left(\frac{P_1(t)}{P_2(t)}\right)^{-\frac{\rho}{1-\rho}} + 1 = (1-\psi)^{-\frac{1}{1-\rho}} P_2(t)^{\frac{\rho}{1-\rho}} \\
\Longrightarrow & -\frac{\rho}{1-\rho}\left(\hat{P}_1(t) - \hat{P}_2(t)\right)\left(1 - (1-\psi)^{\frac{1}{1-\rho}} P_2(t)^{-\frac{\rho}{1-\rho}}\right) = \frac{\rho}{1-\rho}\hat{P}_2(t) \\
\Longrightarrow & -\left(\hat{P}_1(t) - \hat{P}_2(t)\right)\psi^{\frac{1}{1-\rho}} P_1(t)^{-\frac{\rho}{1-\rho}} = \hat{P}_2(t)
\end{aligned}
$$

Equation (10) gives the expression of $\hat{P}_1(t)-\hat{P}_2(t)$ in terms of $\hat{w}$. $\hat{P}_1(t)$ is determined likewise.

Appendix A.2. Proof of Equation (16)

I use the labor market clearing condition, and I assume that the labor supply is constant.

$$
\begin{aligned}
& N_1(t) + N_2(t) = N(t) \\
\Longrightarrow & S_1^N(t)\hat{N}_1(t) + S_2^N(t)\hat{N}_2(t) = 0 \\
\Longrightarrow & S_1^N(t)\left(\hat{P}_1(t) + \hat{\epsilon}_1(t)\right) + S_2^N(t)\left(\hat{P}_2(t) + \hat{\epsilon}_2(t)\right) = \left(1 - \alpha_1 S_1^N(t) - \alpha_2 S_2^N(t)\right)\hat{w}(t)
\end{aligned}
$$

Next, I replace $\hat{P}_1(t)$ and $\hat{P}_2(t)$ by their respective expressions in equations (18) and (19).

Appendix A.3. Proof of Equation (15)

$$
\begin{aligned}
& Q(t) = P_1(t)Q_1(t) + P_2(t)Q_2(t) \\
\implies & \hat{Q}(t) = S_1^Q \left(\hat{P}_1(t) + \hat{Q}_1(t)\right) + S_2^Q \left(\hat{P}_2(t) + \hat{Q}_2(t)\right) \\
\implies & \hat{Q}(t) = \frac{1}{1-\gamma}\left(S_1^Q \left(\hat{P}_1(t) + \hat{\epsilon}_1(t) + (\gamma - \alpha_1)\hat{w}(t)\right) + S_2^Q \left(\hat{P}_2(t) + \hat{\epsilon}_2(t) + (\gamma - \alpha_2)\hat{w}(t)\right)\right) \\
\implies & (1-\gamma)\hat{Q}(t) = S_1^Q \left(\hat{P}_1(t) + \hat{\epsilon}_1(t) + (\gamma - \alpha_1)\hat{w}(t)\right) + S_2^Q \left(\hat{P}_2(t) + \hat{\epsilon}_2(t) + (\gamma - \alpha_2)\hat{w}(t)\right)
\end{aligned}
$$

Next, I replace $\hat{P}_1$ and $\hat{P}_2$ by their respective expressions in equations (18) and (19). On the balanced growth path, $(1-\gamma)\hat{Q}(t)$ is constant.

Appendix A.4. Proof of Proposition 3

Under the condition of no structural change, the employment share and the value added share are constant over time. In this case, equation (16) implies that:

$$\hat{w} = x_1\hat{\epsilon}_1 + x_2\hat{\epsilon}_2$$

where x_1 and x_2 are constant numbers. The formal expression of x_1 and x_2 are given by the following:

$$x_1 = \frac{\rho(1-\gamma)S_1^N + (1-\rho)S_1^Q}{\rho(1-\gamma)\left(1 - S_1^N\alpha_1 - S_2^N\alpha_2\right) + (1-\rho)\left(1 - S_1^Q\alpha_1 - S_2^Q\alpha_2\right)}$$

$$x_2 = \frac{\rho(1-\gamma)S_2^N + (1-\rho)S_2^Q}{\rho(1-\gamma)\left(1 - S_1^N\alpha_1 - S_2^N\alpha_2\right) + (1-\rho)\left(1 - S_1^Q\alpha_1 - S_2^Q\alpha_2\right)}$$

Given the definition of structural change, as well as the expressions for the employment share in equation (12) and the value added share in equation (13) , structural change implies that $\hat{\Delta}$ is not equal to zero. Therefore, balanced growth without structural change implies the following:

$$\hat{\Delta} = 0 \implies \hat{\epsilon}_2 - \hat{\epsilon}_1 + (\alpha_2 - \alpha_1)\hat{w} = 0$$
$$\implies \hat{\epsilon}_2 - \hat{\epsilon}_1 + (\alpha_2 - \alpha_1)(x_1\hat{\epsilon}_1 + x_2\hat{\epsilon}) = 0$$
$$\implies \hat{\epsilon}_1 = \chi_0\hat{\epsilon}_2$$

where χ_0 is defined by:

$$\chi_0 = \frac{(\alpha_2 - \alpha_1)x_2 + 1}{1 - (\alpha_2 - \alpha_1)x_1} = \frac{1 - \alpha_1}{1 - \alpha_2}$$

The wage rate can then be expressed in terms of the TFP growth in one sector.

$$\hat{w} = (\chi_0 x_1 + x_2)\,\hat{\epsilon}_2$$

The condition of balanced growth implies the following:

$$\Big(S_1^Q\chi_0 + S_2^Q - \Big(S_1^Q(\gamma - \alpha_1) + S_2^Q(\gamma - \alpha_2)\Big)\,(\chi_0 x_1 + x_2)\Big)\,\hat{\epsilon}_2 \text{ is constant}$$

Thus, since the term in parenthesis is constant, the condition implies that $\hat{\epsilon}_2$, and similarly $\hat{\epsilon}_1$, is constant.

Appendix B. Constant Return to Scale

Lemma 1. *Let (X, d) and (Y, d') be two metric spaces. A function $f : X \to Y$ is continuous at a point $a \in X$ if and only if whenever $\lim_{n\to\infty} a_n = a$ for a sequence $a_1, a_2, ...$ of points of X, $\lim_{n\to\infty} f(a_n) = f(a)$.*

Proof. see ch.2 theorem 5.4 of Mendelson (1990) □

Appendix B.1. Proof of Proposition 4

Since $\mathcal{M}$ and $\mathcal{F}_\gamma$ are continuous then $\mathcal{M} \circ \mathcal{F}_\gamma$ is continuous in γ and $\mathcal{M} \circ \mathcal{F}_\gamma = \Phi_\gamma$, $\forall\gamma < 1$. Let's now consider a given sequence $\{\gamma_n\}_{n\in\mathbb{N}}$ in [0,1) s.t. $\lim_{n\to\infty} \gamma_n = 1$ (for example

$\gamma_n = 1 - \frac{1}{n}, \forall n \in \mathbb{N}^*)$. Using Lemma 1 we have the following:

$$\mathcal{M} \circ \mathcal{F}_1 = \mathcal{M} \circ \mathcal{F}_{\underset{n\to\infty}{lim} \gamma_n} = \underset{n\to\infty}{lim} \mathcal{M} \circ \mathcal{F}_{\gamma_n} = \underset{n\to\infty}{lim} \Phi_{\gamma_n} = \Phi_1$$

Appendix B.2. Proof of Proposition 5

The growth rate of the aggregate output is given in Appendix A.3. It is expressed as a function of γ as follows:

$$\Phi_\gamma = \frac{S_1^Q \hat{\epsilon}_1 + S_2^Q \hat{\epsilon}_2 - \left(S_1^Q(\gamma - \alpha_1) + S_2^Q(\gamma - \alpha_2)\right)\hat{w}}{1-\gamma} \tag{B.1}$$

where $\hat{w}$ is given by equation (16). S_1^Q, S_2^Q, and $\hat{w}$ are all function of γ. In light of Proposition 4, the growth rate of the economy with CRS is equal to the limit of Φ_γ when γ tends to one.

In order to compute the limit, let us notice that the numerator and the denominator are all equal to 0, when $\gamma = 1$. In this case, I use L'Hôpital's rule which is stated in Lemma 2.

Lemma 2. *Let f and g be two differentiable functions on an open interval I. Let $x_0 \in I$, such that $f(x_0) = g(x_0) = 0$, and $g'(0) \neq 0$. Then*

$$\underset{x\to x_0}{lim} \frac{f(x)}{g(x)} = \frac{f'(x_0)}{g'(x_0)}$$

In light of Lemma 2, we have the following

$$\begin{aligned}\Phi_1 = \underset{\gamma\to 1}{lim} \Phi_\gamma &= -\frac{\partial\left(S_1^Q \hat{\epsilon}_1 + S_2^Q \hat{\epsilon}_2 - \left(S_1^Q(\gamma - \alpha_1) + S_2^Q(\gamma - \alpha_2)\right)\hat{w}\right)}{\partial\gamma}\Bigg|_{\gamma=1} \\ &= \frac{\rho}{1-\rho}\left(S_1^Q - S_1^N\right)(\hat{\epsilon}_1 - \hat{\epsilon}_2) + \left[\frac{\rho}{1-\rho}\left(S_1^Q - S_1^N\right)(\alpha_1 - \alpha_2) + 1\right]\hat{w}\end{aligned}$$